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REMOTE STORAGE

PRODUCTION OF LIGHT BY CANAL RAYS

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THE PRODUCTION OF LIGHT BY CANAL RAYS

BY GORDON SCOTT FULCHER

The two phenomena of greatest significance in connection with canal rays are well known to be the simultaneous electrostatic and magnetic deflection of the rays, first obtained by W. Wien, and the Doppler effect, which J. Stark discovered to be shown by light from the path of the canal rays. The Wien experiment enables us to determine the velocity and mean value of $\frac{e}{m}$ of the constituent rays; the Stark effect gives us directly the speed in the line of sight of the sources of the light showing the effect. Two years ago, from a summary and comparison of the experimental results reported by W. Wien, J. J. Thomson, J. Stark, F. Paschen, and others,¹ it seemed evident that the distribution of velocities among the canal rays is quite different from that among the sources of the light showing the Stark effect. It appeared to be necessary to conclude that the canal rays cannot themselves be the chief sources of the light in question, as has been generally assumed.²

THE HYPOTHESIS

An alternative hypothesis was suggested. The only other molecules present with velocities great enough for them to be the sources sought are those gas molecules which have been hit by the canal rays and thus have acquired a velocity great or small according to the squareness of the collision. If we suppose this molecular phenomenon to obey the laws of ordinary impact between solid bodies, the momentum given the gas molecule will vary up to a maximum value of the same order as that of the bombarding ray, depending on the relative masses of the two molecules and on the coefficient of restitution. Will the Doppler effect to be expected if these hit molecules emit light agree in all essential

¹ G. S. Fulcher, "Our Present Knowledge of Canal Rays," *Smithsonian Misc. Collections*, **52**, 295-324, 1909.

² *Ibid.*, p. 322.

details with the Stark effect as observed? A statistical computation, explained in detail below (p. 41), showed that a satisfactory agreement was obtained if the following assumptions were made:

1. That the intensity of the light emitted as a result of each impact is proportional to the energy transmitted to the hit molecule, but

2. That the hit molecule emits no light unless the energy so transmitted exceeds a certain minimum supposed to be equal to that necessary to produce ionization; and

3. That the hitting molecules emit no light of the kind showing the Stark effect as a result of the collision. Of these, the first has since been experimentally verified (see below), the second is rendered probable by other experimental evidence, and the third, rather more difficult to accept, will be shown later to be a necessary assumption which perhaps can be made more plausible by the consideration that most of the hitting charged rays may be neutralized by the collisions which ionize the hit molecules. Hence if the hitting molecules thus neutralized emit any light, it is not the series-line spectrum which is emitted by the ionized molecules and which alone shows the Stark effect.

It is possible then to reconcile the results of the deflection experiments with the Stark effect if we assume that the sources of the light showing this effect are the gas molecules hit and ionized by the canal rays rather than the canal rays themselves.¹

EXPERIMENTAL EVIDENCE

If this hypothesis is correct the intensity of the light from a canal-ray beam should vary directly as the number of collisions per unit time, per unit length of path, that is, with the pressure of the gas, providing the number and velocity of the canal rays are kept constant.

The following apparatus, shown diagrammatically in Fig. 1, has been designed to test this deduction. It is so arranged that the pressure of the gas in the canal-ray chamber back of the cathode can be varied in the ratio of one to twenty without changing the pressure in the discharge chamber, which determines the

¹ *Ibid.*

cathode-fall of potential, that is, the number and velocity of the canal rays. In one experiment hydrogen gas from the reservoir where the pressure is maintained at a certain constant value of about 5 cm of mercury, passes slowly and continuously through

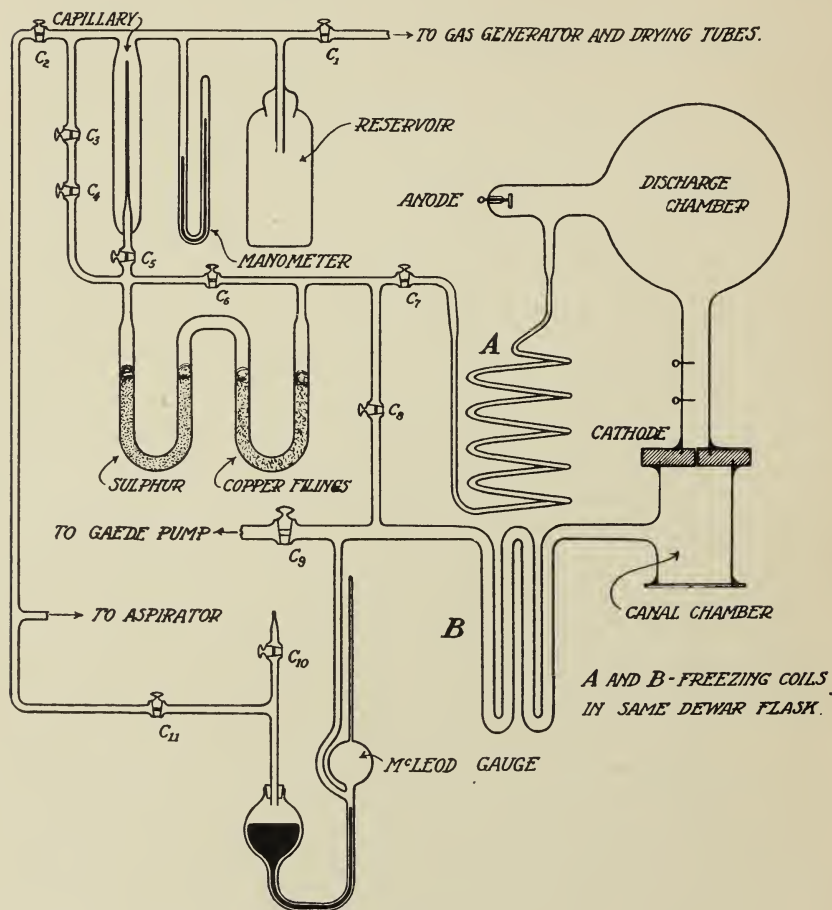


FIG. 1

a capillary tube (10 cm long and with a bore about 0.009 cm in diameter) and some purifying tubes to the discharge chamber. The pressure here is about 0.1 mm of mercury. From this chamber a single hole in the aluminum cathode (0.05 cm in diameter and 1 cm long) leads to the canal chamber, which is

maintained at a pressure of about .005 mm of mercury by a Gaede mercury pump to which it is connected by large tubes. The hole in the cathode thus behaves like a porous plug of a single pore. After equilibrium is established, these pressure relations can be maintained indefinitely, and the intensity of the light emitted when the canal rays pass through gas at the low pressure maintained in the canal chamber can be measured at leisure.¹

On the other hand the pump may be stopped and the pressure allowed to become the same on both sides of the cathode and equal to the pressure formerly existing only on the discharge-chamber

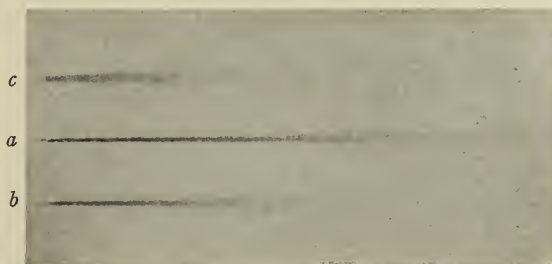


FIG. 2.—Photographs of the canal-ray beam
a, Beam through gas at low pressure (0.005 mm)
b, Beam through gas at higher pressure (0.1 mm)
c, Extra-focal image of beam, higher pressure

side. Conditions in the discharge chamber will now be the same as in the first experiment; hence the current and the cathode-fall of potential, and therefore the number and velocity of the canal rays, will be the same in both cases, though the pressure in the canal chamber is quite different. Hence the intensity of the light from the canal-ray beam should vary in the same ratio as the pressure of the gas in the canal chamber, if our hypothesis is correct.

To measure the intensity of the light, the beam was photographed with an ordinary camera. The time of exposure was adjusted so that in all cases images of about the same density were obtained. The camera was thrown slightly out of focus so as to secure broader images whose density could be more easily compared

¹ To avoid confusion, the discussion of certain details of the apparatus is postponed to the end of the paper.

by the use of a spectrophotometer (see Fig. 2). The intensity of the light was assumed proportional to the density of the image divided by the time of exposure. The pressure was measured with a McLeod gauge connected to the canal chamber. The results for two cathode-falls of potential are given in the following table.

TABLE I

Cathode-Fall Volts		Pressure mm of Hg	Time of Exposure t	Density of Image D	Intensity of Light $I = \frac{D}{t}$	$\frac{I_1}{I_2}$	$\frac{p_1}{p_2}$	$\frac{\frac{I_1}{p_1}}{\frac{I_2}{p_2}}$
3900	$\begin{cases} 1 \\ 2 \end{cases}$	$\begin{matrix} 0.102 \\ 0.0053 \end{matrix}$	$\begin{matrix} 60 \text{ sec.} \\ 1200 \text{ ''} \end{matrix}$	$\begin{matrix} 0.41 \\ 0.47 \end{matrix}$	$\begin{matrix} 0.0068 \\ 0.00039 \end{matrix}$	$\left\{ \begin{matrix} 17.5 \\ 19.2 \end{matrix} \right\}$	$\begin{matrix} 17.5 \\ 19.2 \end{matrix}$	$\begin{matrix} 0.91 \\ 0.91 \end{matrix}$
4400	$\begin{cases} 1 \\ 2 \end{cases}$	$\begin{matrix} 0.096 \\ 0.0044 \end{matrix}$	$\begin{matrix} 50 \text{ sec.} \\ 1200 \text{ ''} \end{matrix}$	$\begin{matrix} 0.44 \\ 0.47 \end{matrix}$	$\begin{matrix} 0.0088 \\ 0.00039 \end{matrix}$	$\left\{ \begin{matrix} 22.6 \\ 21.9 \end{matrix} \right\}$	$\begin{matrix} 22.6 \\ 21.9 \end{matrix}$	$\begin{matrix} 1.03 \\ 1.03 \end{matrix}$

If the hypothesis is correct $\frac{I_1}{p_1}$ should be approximately equal to $\frac{I_2}{p_2}$. The agreement, though unsatisfactory, is within experimental errors.

It is here assumed that light of a certain intensity acting for twenty minutes will produce the same density of image as light of twenty times the intensity acting for one minute. To test this, half a plate was exposed to light from a point source at a distance of 125 cm for 45 seconds and the other half was exposed for 720 seconds at a distance of 500 cm from the same source. Here the ratio of intensities is 16 and equal to the inverse ratio of the times of exposure. No difference can be observed between the densities of the two halves of the developed plate—proving that for the intensities used the assumption is sufficiently accurate.

I gladly acknowledge my indebtedness to Professor C. E. Mendenhall for help in designing the apparatus. The use of a capillary in combination with a Gaede pump to secure equilibrium conditions and thus make the experiment quantitative was suggested by him.

The conclusion from this experiment is, therefore, that the intensity of the light from the canal-ray beam is proportional to

the pressure of the gas through which the canal rays pass, that is, to the probable number of collisions per unit length of path, per unit time, between the canal rays and gas molecules, provided that the number and velocity of the canal rays is unchanged.¹

Next the question arises, how does the intensity of the light emitted vary with the velocity of the rays? Does it vary directly as the total energy-flux of the rays times the gas pressure, that is,

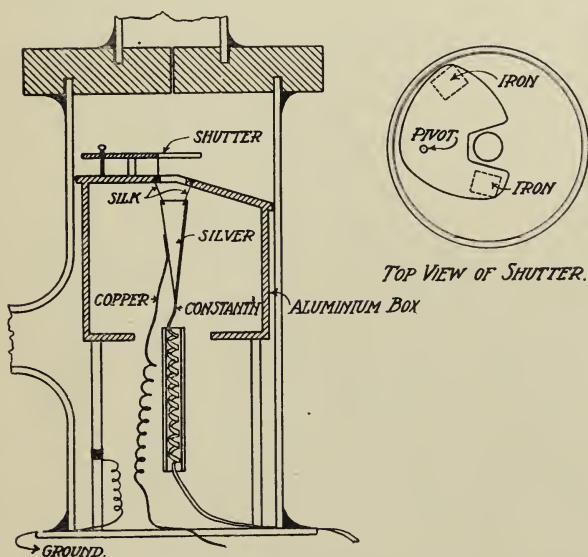


FIG. 3.—Apparatus for measuring the energy-flux of the canal rays

as the mean energy of each ray times the number of collisions per unit length of path per second? To answer this it became necessary to measure the energy-flux of the rays as a function of the cathode-fall of potential. To this end, they were allowed to strike inside a light silver cone so that their kinetic energy might be transformed to heat energy and measured from the temperature changes produced.

In the canal chamber the apparatus shown in Fig. 3 was placed. The silver cone weighing only a fifth of a gram (3 cm long and 0.5 cm in diameter at its base) was suspended in the path of the rays,

¹ This result was reported at a meeting of the Am. Phys. Soc., November 27, 1909.

by means of silk threads, from a grounded aluminum box which served to shield the cone from sudden changes of temperature. To measure the instantaneous temperature of the cone a thermocouple was used. Along one side of the cone was soldered a constantan wire rolled flat, along the other a copper wire. The other junction (see Fig. 4) was maintained at a constant adjustable temperature by immersion in kerosene in a Dewar flask provided

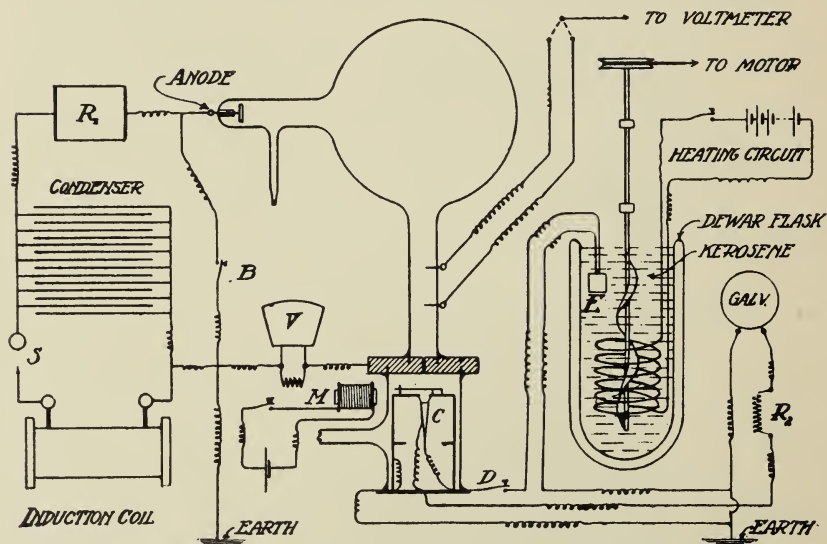


FIG. 4.—Diagram showing the electrical connections of the apparatus

with a heating coil and stirrer.¹ To measure the thermo-electric current a Thomson galvanometer was used whose sensitiveness of 4×10^{-9} amperes per mm deflection could be decreased by introducing resistance *R*₂. For the most sensitive adjustment used (*R*₂=0), 1 mm deflection corresponded to about 0.001°C increase in temperature of the cone or a net addition to the cone of 1.2×10^{-5} calories. Two telescopes were mounted so that through one the observer could follow the second hand of a watch while noting the galvanometer reading with the other eye. Thus a series of observations at intervals of 5 seconds could be made and

¹To avoid confusion, the discussion of other details of the apparatus is postponed to the end of the paper.

the instantaneous temperature of the cone as a function of the time determined. Typical curves are shown in Fig. 5. On starting the discharge through the tube the temperature rises rapidly at first, then more and more slowly. The curve never becomes horizontal because of the radiation received from the back surface of the cathode, which increases with the time as the cathode is

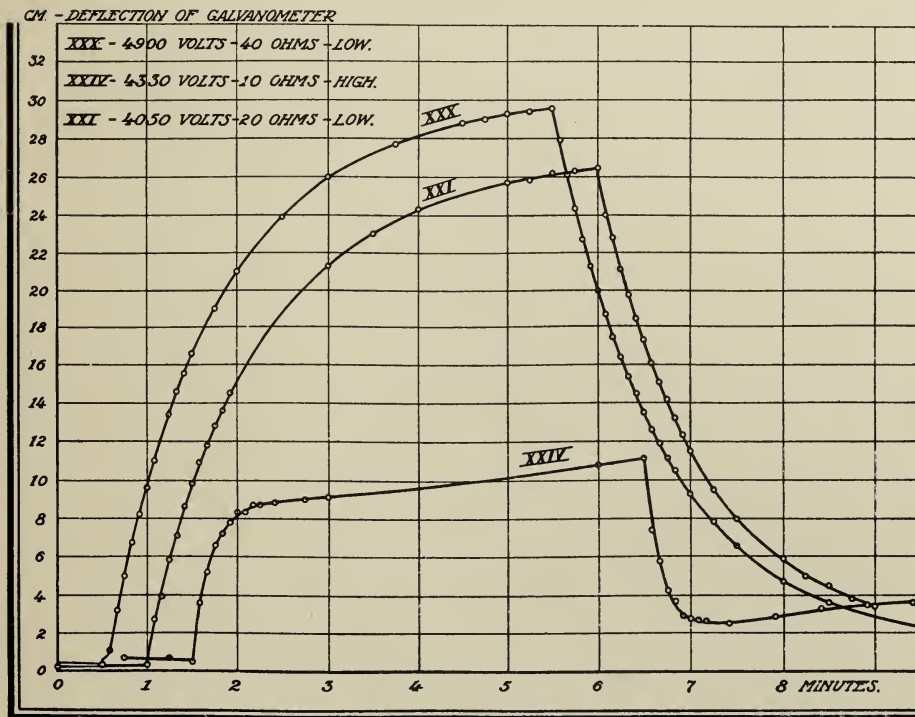


FIG. 5.—Temperature of the cone as a function of the time

heated by the discharge. At the instant when the discharge is stopped, the quickness with which rapid cooling begins is noteworthy. Curves XXI and XXX were taken when the pressure in the canal chamber was low (0.005 mm), while curve XXIV was obtained with a much higher pressure (0.1 mm). The sensitiveness is twice as great for XXIV as for XXI and that in turn twice as great as for XXX.

The problem now is to determine the value of the energy-flux

corresponding to each of these curves. It is easily shown that the temperature is in no case a simple exponential function of the time, as would be the case if the heating of the cone by the rays and the radiation of heat to a constant temperature envelope alone were involved. The radiation from the heated cathode is an important disturbing factor. If we let

- x = temperature of the cone,
- y = temperature of the surrounding gas and case,
- z = temperature of back side of the cathode,
- c = heat capacity of the cone,
- E = energy received by the cone from the canal rays per second,
- A and B = radiation constants, and
- C = convection constant depending on the gas pressure,

then

$$c \frac{dx}{dt} = E + A(z - x) - (B + C)(x - y) \quad (1)$$

Just preceding the instant ($t=5$) when the discharge is discontinued by short-circuiting the tube we have

$$c \left(\frac{dx}{dt} \right)_{-5} = E + Az_5 + (B + C)y_5 - (A + B + C)x_5 \quad (2)$$

while immediately following the cessation of the discharge

$$c \left(\frac{dx}{dt} \right)_{+5} = Az_5 + (B + C)y_5 - (A + B + C)x_5 \quad (3)$$

since there is no discontinuity at that instant of either x , y , or z . Hence

$$E = c \left(\frac{dx}{dt} \right)_{-5} - c \left(\frac{dx}{dt} \right)_{+5} \quad (4)$$

an equation which serves most readily for the determination of E from the curves.

The difficulty then was to determine $\left(\frac{dx}{dt} \right)_{+}$ accurately, since the cooling was so rapid at first that the galvanometer readings were uncertain immediately after the discharge was stopped. The method employed was a graphical one. A smooth curve was drawn through the galvanometer readings plotted as a function of the time (Fig. 5), making due allowance for the fact that the

current carried by the rays, which in part passed through the galvanometer, ceased at the same instant as the discharge. The ordinates of the curve were read at regular intervals of time and the values of the time derivatives thus obtained were plotted as a

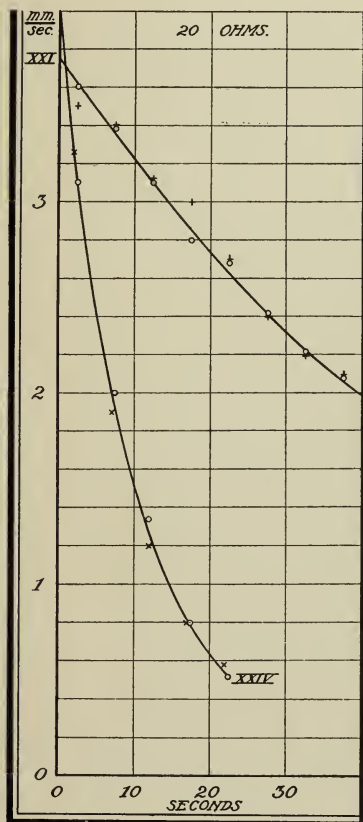


FIG. 6

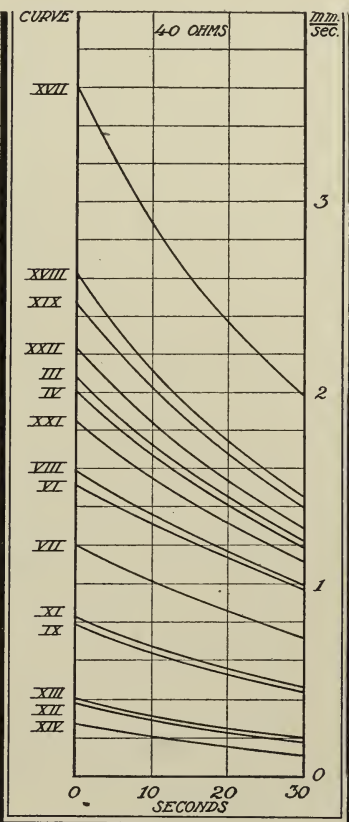


FIG. 7

Time derivative of the temperature of the cone as a function of the time

function of the time. Fig. 6 shows two such derivative-time curves corresponding to two of the temperature-time curves of Fig. 5. The circles were obtained from the cooling part of the curves, the crosses from the heating part. The former are the more reliable. It is seen that a smooth curve can be drawn through the points with considerable certainty for XXI (low pressure)

and the value of the derivative for $t=0$ can be determined with some accuracy. A number of such derivative-time curves are shown plotted to the same scale in Fig. 7. These curves were each drawn independently. The fact that they fit in with each other so well is evidence for their accuracy.

The results obtained for the energy-flux of the canal rays as a function of the cathode-fall of potential are plotted in Fig. 8. It

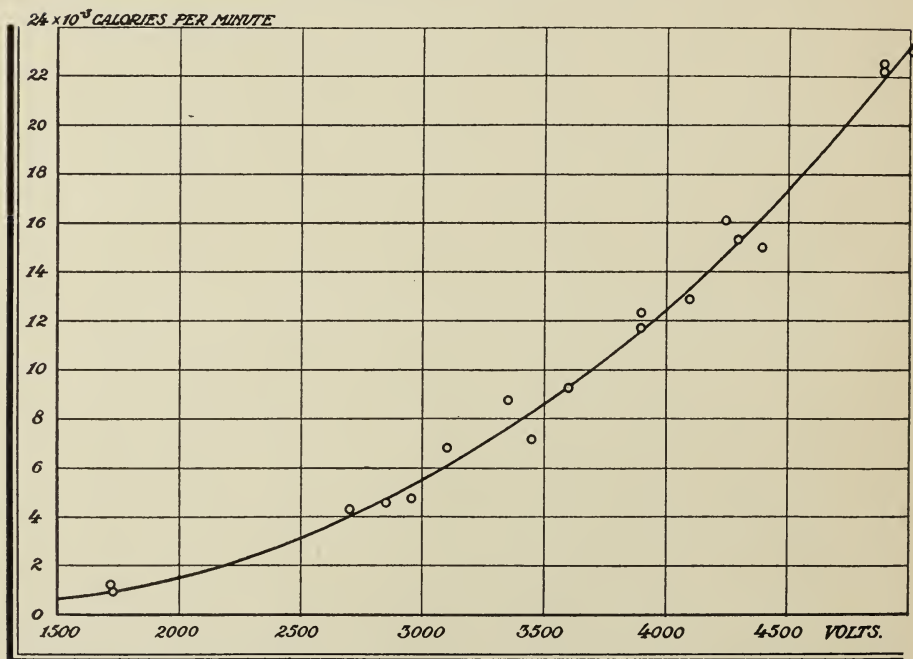


FIG. 8.—Energy-flux of canal-ray bundle as a function of the cathode-fall of potential

is to be regretted that the agreement between the various points obtained is not better. The trouble does not lie in the determination of the energy-flux from the curves, since that is probably accurate to a few per cent, but rather in the measurement of the cathode-fall of potential. The Kelvin electrostatic voltmeter could be read to 1 per cent, but even though special precautions were taken, as will be described later, to secure a continuous discharge through the tube, the cathode-fall of potential was seldom quite constant, so that readings could not be made as accurately as

otherwise. The effect of slight impurities in the gas, such as were doubtless given out by the cathode during the discharge, is very marked, though in no cases were these impurities sufficient to affect the spectrum to any noticeable extent. It is seen that a change of less than 5 per cent in the abscissas of the points plotted in Fig. 8 would bring all of them on the curve drawn through them. We

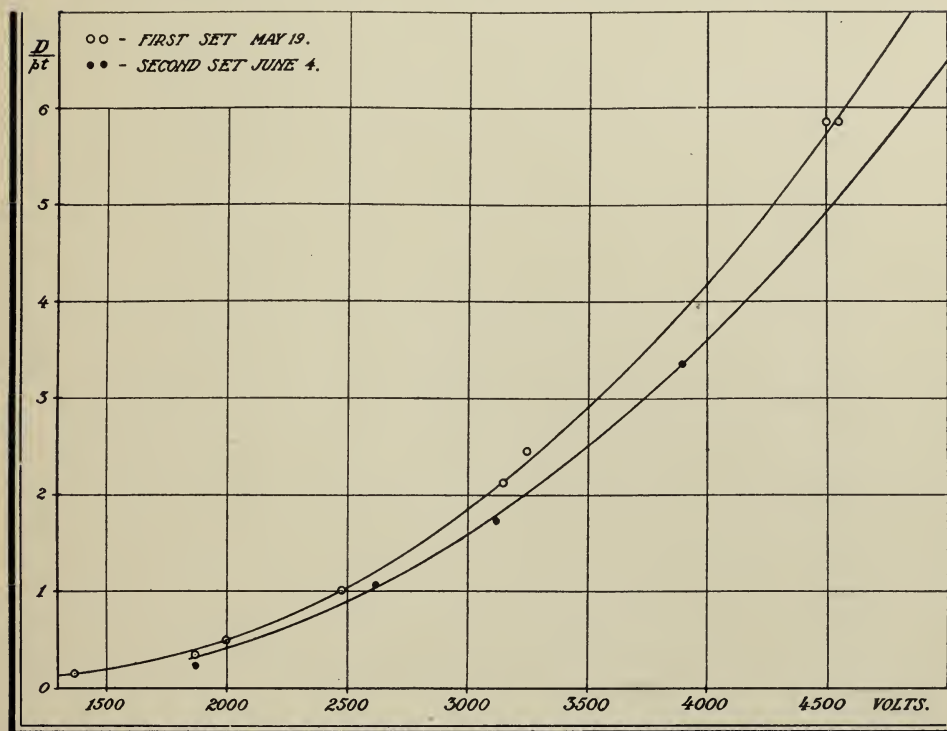


FIG. 9.—Ratio of light-intensity to gas pressure as function of cathode-fall of potential

must conclude, therefore, that the curve represents the experimental results as closely as possible and that the deviations are less than possible experimental errors.

The variation of the intensity of the light from the canal-ray beam as a function of the cathode-fall of potential was determined precisely as in the first experiment by photographing the beam, adjusting the times of exposure so as to obtain images of about the same density. The results are plotted in Fig. 9. The ordinates

are proportional to $\frac{D}{pt}$, where D is the density of the image measured with a spectrophotometer, p is the gas pressure, and t the time of exposure. Two sets of determinations were made. The two curves drawn through the points obtained are each identical with the curve shown in Fig. 8 except for a constant factor of proportionality. The ordinates of Fig. 8 are proportional to the mean energy of the individual rays times the number of rays striking the cone per second when the pressure in the canal chamber is low; that is, they are very closely proportional to the mean energy of the individual rays times the number of rays emerging from the hole in the cathode per second when the pressure is the same on both sides of the cathode. The ordinates of the curves in Fig. 9 are proportional to the mean intensity of the light emitted as a result of each collision times the number of rays emerging from the hole in the cathode per second when the pressure is the same on both sides of the cathode. The fact that through the range of cathode-falls used these curves agree shows that the mean intensity of light emitted per collision is proportional to the mean energy of the individual rays, that is, to the mean energy of each collision. It is not to be expected that this law holds rigorously, and deviations may be expected to increase as the minimum voltage necessary to produce a discharge is approached; but within the limits of voltage and velocity used here (1,500 to 5,000 volts, 6 to $10 \times 10^7 \frac{\text{cm}}{\text{sec}}$) the law seems to be verified within the limits of experimental error, that is, within a few per cent.

DISCUSSION AND STATISTICAL CALCULATIONS

What bearing have these experimental results upon the theory of the production of the light in question? First, it must be pointed out that in the case of canal rays in pure hydrogen the light producing the displaced lines in the Stark effect is several times as intense as that producing the rest lines, hence the experimental results which apply strictly only to the whole of the light from the path of the rays may be taken without serious error to apply to the light from the moving sources alone. The experi-

ments therefore tend to prove that the light showing the Stark effect is emitted only as a result of the collisions of canal rays with gas molecules and that the intensity of the light emitted per collision is proportional to the mean energy imparted to the hit molecules by the collisions.

Can the details of the Stark effect be explained on this basis? If so, what additional assumptions are necessary? In answering these questions a calculation made three years ago, the results of which were reported in part in the article referred to above, is of some importance. By a statistical method a computation was made of the Stark effect to be expected if the following assumptions are true:

1. That canal rays all with the same velocity enter a gas whose molecules are identical with the rays and have velocities negligible in comparison with that of the rays;

2. That the momenta after collision are the same as if the colliding molecules were perfectly elastic spheres;

3. That the intensity of the light emitted by each hit molecule is proportional to the kinetic energy given it as a result of the collision, but

4. That no light is emitted by a hit molecule unless the energy transmitted to it exceeds a certain minimum, which is supposed to be equal to the energy necessary to produce ionization; and finally

5. That no light of the kind in question is emitted by the hitting rays—the supposition being that they are for the most part neutralized by the collisions, hence do not emit the same spectral lines as the ionized hit molecules.

Of these assumptions the first two are made for the sake of simplicity. Their disagreement with the actual facts in the case of hydrogen rays in hydrogen gas is probably not sufficient to affect the qualitative value of the computation. The third assumption has since been proved by the experiment described above. The last two are equivalent to the single assumption that the series-line spectrum is emitted only by the molecules which become positively charged as a result of the shock of the collision. To find out whether this assumption is necessary to explain the details

of the Stark effect, as reported by J. Stark, F. Paschen, and B. Strasser, on the basis of the emission of light only as a result of molecular collision (necessitated by the first experiment above), is the purpose of the following computations.

The method employed was necessarily statistical, as the problem is too complicated to yield to direct mathematical treatment. Using the second assumption, we can readily determine the velocity

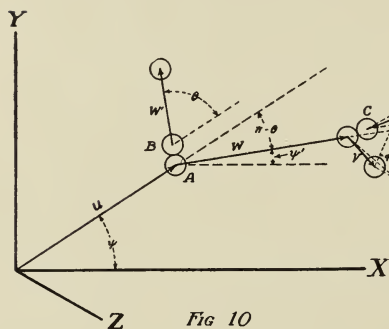


Fig 10

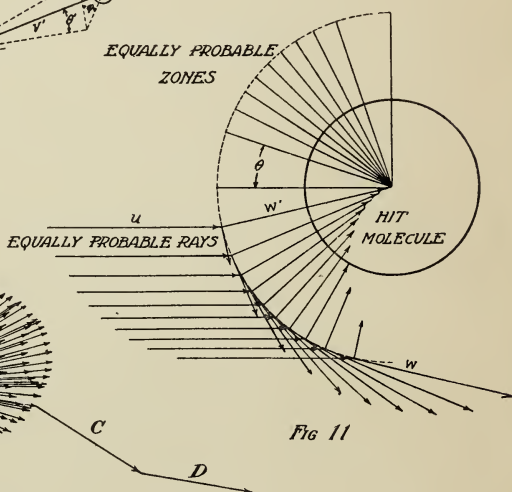


Fig 11

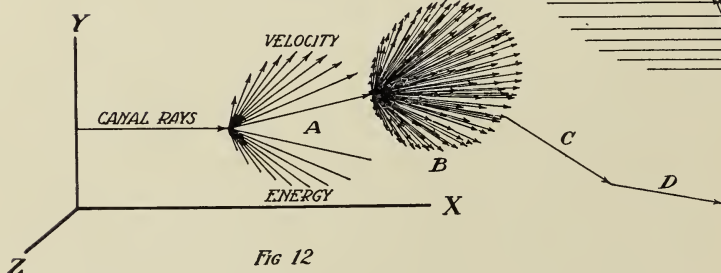


Fig 12

of the hit and of the hitting molecule after each impact. If a molecule with velocity u (Fig. 10) hits a molecule at rest so that the line of centers at the instant of collision lies in the XY plane and makes an angle θ with u , the resulting velocities are:

$$\begin{aligned} \text{(for hit molecule)} \quad W' &= u \sin \theta; \quad W'_x = W' (\cos \theta \cos \psi - \sin \theta \sin \psi) \\ \text{(for hitting molecule)} \quad W &= u \cos \theta; \quad W_x = W (\sin \theta \cos \psi + \cos \theta \sin \psi) \end{aligned}$$

If the line of centers is not in the XY plane but its projection on a plane perpendicular to the velocity of the hitting molecule before collision (W) makes an angle ϕ with the XY plane, and if

θ' is the angle it makes with W and ψ' is the angle W makes with the X -axis, then after the collision,

$$\begin{aligned} \text{(for hit molecule)} \quad V' &= W \sin \theta'; \\ V'_x &= V'(\cos \theta' \cos \psi' - \sin \theta' \sin \psi' \cos \phi). \\ \text{(for hitting molecule)} \quad V &= W \cos \theta'; \\ V_x &= V(\sin \theta' \cos \psi' + \cos \theta' \sin \psi' \cos \phi). \end{aligned}$$

Now consider Fig. 11. The center of the hitting molecule at the instant of collision must lie on the dotted spherical surface. Suppose its velocity is u . It can readily be shown that if the

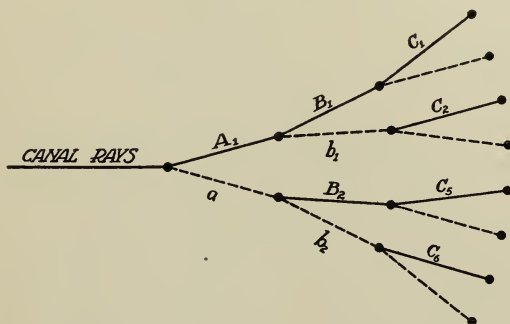


FIG. 13.—Various groups of secondary rays

spherical surface is divided into zones by coaxial cones whose angular apertures are given by the equations $\cos^2 \theta = \frac{a}{10}$, where a is an integral parameter varying from 1 to 10, the probability that the center of the hitting molecule will lie within any one zone is the same since the areas of projection of the zones on a plane perpendicular to u are all equal. It is further evident that if each zone is divided into two equally probable zones, the values of θ corresponding to the dividing cones will be equally probable values of θ . If then we assume ten collisions for a given value of u such that the line of centers at collision makes angles with u given by the equation $\cos^2 \theta = \frac{1}{20} + \frac{a}{10}$, where a has values 0, 1, 2, . . . 8, 9, the resulting velocities after collision may be taken to represent fairly the actual distribution of velocities as far as θ is concerned.

These equally probable rays with the velocities of the hit and hitting molecules after collision in each case are shown in Fig. 11.

The general method of treating the problem in hand is shown in Fig. 12. Canal rays are assumed to enter the gas all moving with the same velocity u in the direction of the X -axis, which is also the line of sight. Let each ray collide with a gas molecule. The distribution of velocities among this first generation of secondary rays (A) can be fairly represented for our purpose by the ten

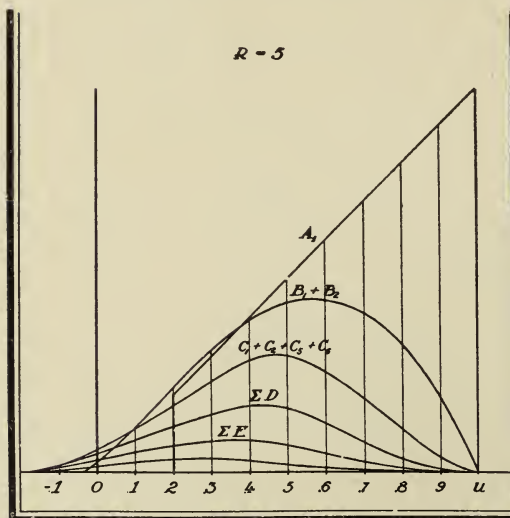


FIG. 14.—Doppler effect due to the various generations of secondary rays

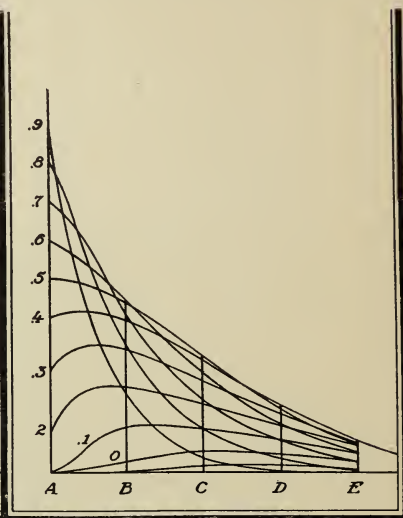


FIG. 15.—Extrapolation curves

bundles of rays shown in the XY plane, since the Doppler effect depends in this case only on the latitude angle θ and not on the azimuth angle ϕ . The energy given the hit molecules is shown in the lower half of the same diagram. It is seen that by far the most light is emitted by the molecules having a considerable velocity in the line of sight. Let us assume that the minimum energy necessary to produce ionization is some fraction, say one-fifth of the original energy of each canal ray. In that case two of our bundles of hit molecules will emit no light.

Now let each of these secondary rays strike gas molecules and produce another generation (B). Here the Doppler effect will

depend on both ϕ and θ ; hence to represent this generation 1,000 bundles of hit molecules were chosen, ten values of ϕ (9° to 171° at intervals of 18°) for each value of θ and ten values of θ for each value of ψ , the angle which the hitting molecules made with the X -axis before collision. One hundred only are shown in perspective in the drawing. The energy and the x component of the velocity of each of these bundles was computed and the energy (taken equal to the square of the velocity) of all those whose

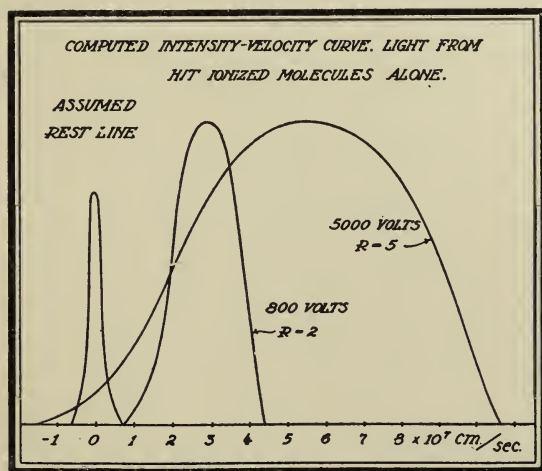


FIG. 16

velocity along the X -axis lay between certain limits, say 0.50 to 0.55 u , was added up and this sum was taken as proportional to the intensity of light corresponding to that Doppler shift. Thus the intensity of light as a function of the Doppler shift to be expected from this generation was obtained (B_1). In getting the effect of the third generation of hit molecules a multiplication by 100 was avoided by further extending the principle of representation. The cosine of ψ' , the angle which each of the 1,000 rays of the second generation made with the X -axis, was computed by dividing the velocity of each ray along the X -axis by its actual velocity. Then these values of the cosine were plotted as a function of the velocity, one point for each ray. These were divided into groups of 20 and one ray picked out to represent each 20.

The values of the velocity and of the angles ψ' for each of these 50 representatives were obtained directly from the diagram. As in the case of B , ten values of ϕ for each of ten values of θ for each ray had to be considered, making 5,000 in all, for each of which the energy and velocity along the X -axis were computed. The intensity of light as a function of the Doppler shift was then obtained as before (C_1).

We have considered so far only the secondary rays in one direct

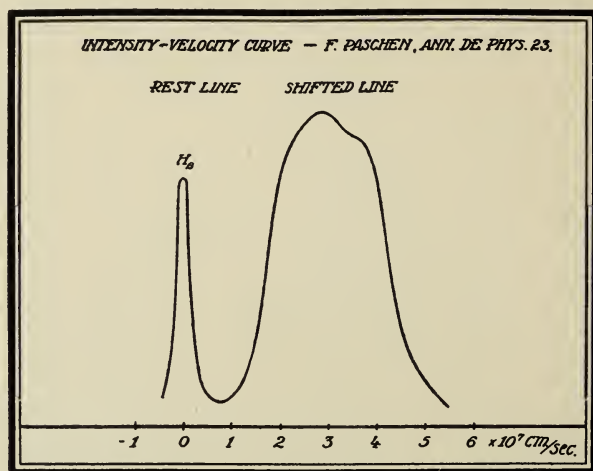


FIG. 17.—Stark effect for hydrogen

line of descent, both in the emission of light and in the production of other secondary rays. But surviving each generation of collisions there are in addition to the ionized hit rays some positively charged hitting rays which failed to ionize the molecules they hit but were merely deflected. In the diagram (Fig. 13), the full lines represent hit ionized molecules which emit light; the dashed lines represent ionized hitting molecules, emitting no light but capable of producing other ionized secondary rays which may emit light. In Fig. 21 the Doppler effect due to each of these groups of rays, on the basis of the assumptions made at the beginning, is shown (A_1 , B_1 , B_2 , C_1 , C_2 , C_5 , C_6) for the special case when the energy of each canal ray is taken to be five times the minimum energy necessary to produce ionization ($R=5$). The total Doppler effect

for each generation for the same case is shown in Fig. 14. The distribution of the light due to the later generations was obtained by extrapolation, using the curves shown in Fig. 15. The sum of them all shows discontinuities at $0.2u$ and u due to our artificial assumption that all the canal rays have the same velocity and that the unshifted line has no width. By modifying these assumptions to agree more closely with the actual facts the discontinuities are eliminated. The results for the cases when $R=2$

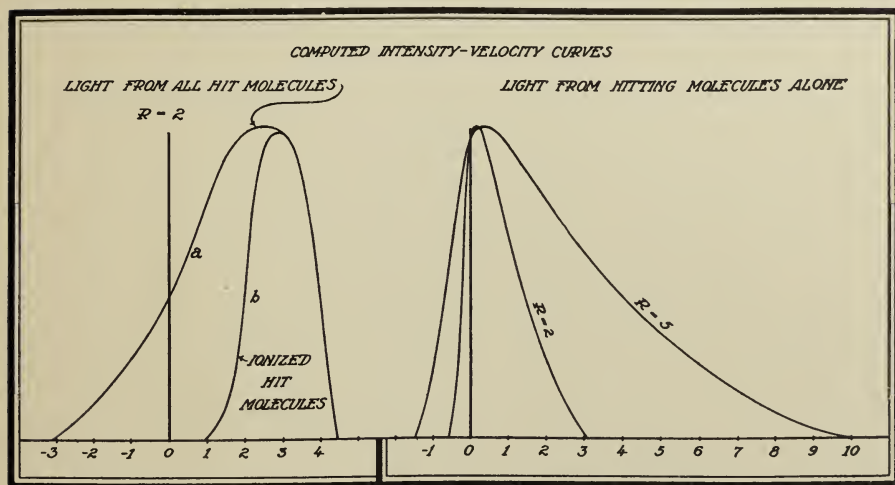


FIG. 18.—Testing the fourth assumption

FIG. 19.—Testing the fifth assumption

and $R=5$ are given in Fig. 16. It was assumed in getting the scale of abscissas that the minimum energy necessary to produce ionization corresponds to a velocity of 2×10^7 cm per sec. in the case of hydrogen. For comparison with the Stark effect as actually observed, an intensity-velocity curve has been reprinted from Professor F. Paschen's paper¹ (Fig. 17). This curve is obviously the sum of two, each having a marked intensity minimum and agreeing in general form with the two computed curves shown in Fig. 16. A better agreement could hardly be expected.

To show the necessity of introducing the fourth assumption regarding the minimum energy necessary to produce ionization,

¹ *Annalen der Physik*, 23, 250, 1907.

a calculation was made without this assumption. The result for the case when $R=2$ is shown in Fig. 18 (a). Curve (b) is reproduced from Fig. 16 for comparison. Evidently the presence of the intensity minimum demands this fourth assumption.

The fifth assumption, that no hitting rays emit light as a result of the collisions, seems rather arbitrary and it appeared desirable

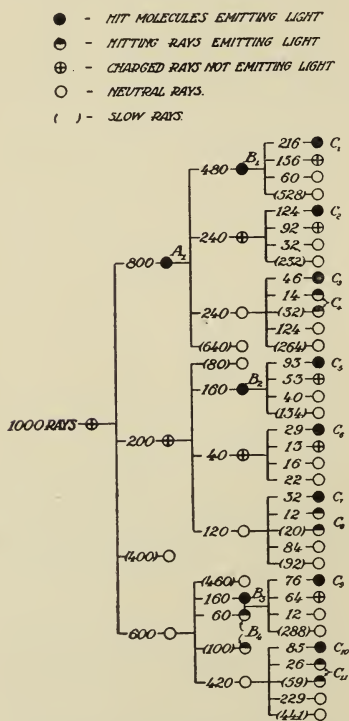


FIG. 20.—Genealogy on the basis of the six assumptions

to determine to what extent this assumption is necessary. If we assume that each of the charged hitting molecules emits light whose intensity is proportional to the energy of the collision, a calculation of the Doppler effect due to this light gives curves shown in Fig. 19. Here again the presence of the intensity-minimum in the Stark effect proves that, for the most part at least, the charged hitting rays do not emit any light corresponding to the series-line spectrum as a result of the collisions.

There is still a further possibility to be investigated. Neutral rays when moving with sufficient velocity doubtless have the power of producing ionization. If so, the hitting molecule is as likely to be ionized as the hit molecule.

Nothing is known as to the mini-

imum energy necessary to produce ionization in this case, but a calculation was made adding the following assumption to the five considered above:

6. That when the collision of a neutral ray with a neutral gas molecule involves the transference of more than a certain minimum energy, which is assumed to be equal to the minimum energy necessary for ionization in the case of the collision of a charged

ray with a gas molecule, one will be ionized so that on the average half the hitting and half the hit molecules will emit light.

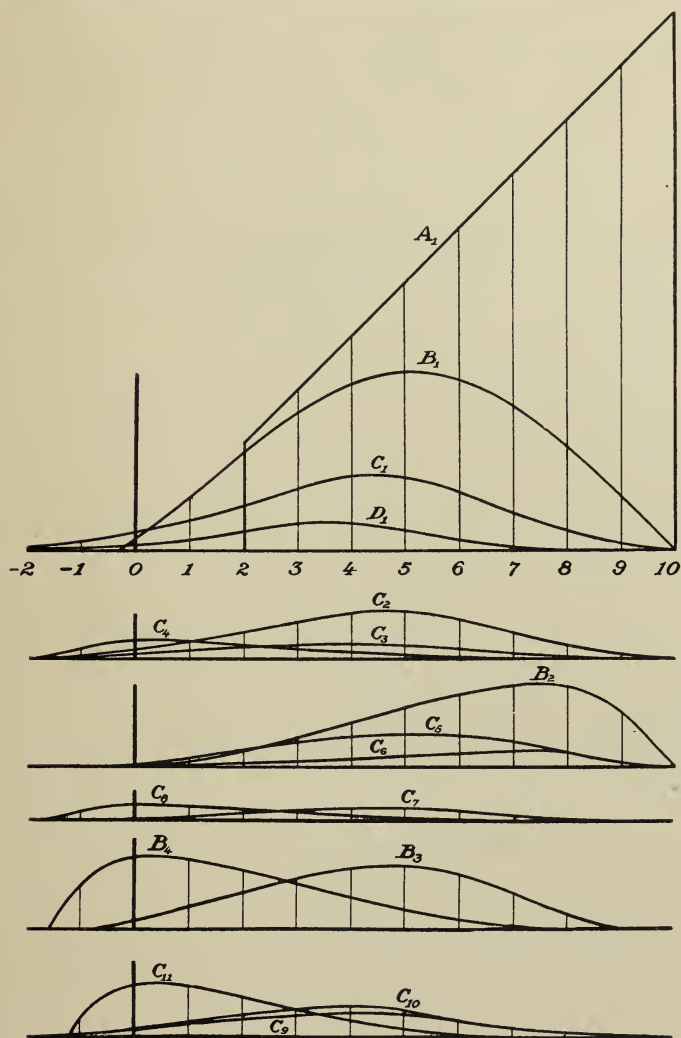


FIG. 21.—Doppler effect due to various groups of secondary rays

The introduction of this assumption adds several groups of active rays to our family. The genealogy for the case of 1,000 positively charged rays ($R=5$) is given in Fig. 20. The slow rays

are those incapable of producing ionization. The Doppler effect corresponding to each of the active groups of rays was computed for the first three generations. The curves are shown in Fig. 21. The effect of later generations was obtained by extrapolation as above (cf. Fig. 15). The total effect due to 1,000 charged canal rays on the basis of the six assumptions is shown in Fig. 22 (a) for the special case when $R=5$. Curve (b) is reproduced from Fig. 16 for comparison. Evidently our sixth assumption is incom-

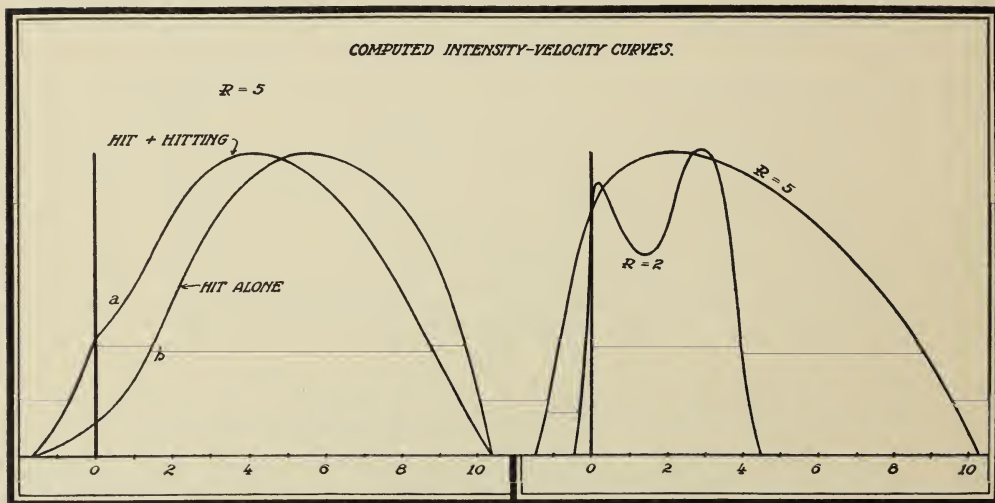


FIG. 22.—Testing the sixth assumption: charged canal rays

FIG. 23.—Doppler effect due to neutral canal rays on the basis of the sixth assumption

patible with an intensity minimum. Probably the minimum energy required for neutral rays to produce ionization is much greater than was assumed and hence the disturbing effect of these neutral rays is much less. Finally a computation was made of the Doppler effect to be expected on the basis of the six assumptions if 1,000 neutral canal rays with the same velocity u enter a gas. The genealogy in this case is similar to that shown in Fig. 20 except that the number in the various groups is quite different. The total Doppler effect due to these 1,000 neutral rays and their offspring is shown in Fig. 23 for the special cases when $R=2$ and

$R=5$. Here again it is evident that the number of neutral rays must be small or assumption 6 must be incorrect. The number of neutral rays, however, is known from deflection experiments not to be small, may be in fact over 30 per cent of the whole number. The other alternative is therefore inevitable.

CONCLUSIONS

As a result of these calculations one seems justified in concluding that the original assumptions regarding the minimum energy necessary to produce the emission of light and the emission of light by the hit ionized molecules alone, represent the true state of the case very approximately. If the neutral rays produce ionization, the minimum energy required must be considerably greater than is necessary in the case of charged rays, so that the light thus produced is insignificant. The hitting charged rays must be neutralized for the most part and particularly when the shock of collision is great, at the same instant as the hit molecule is ionized.

If this analysis of the phenomenon is correct, a far more important conclusion necessarily follows: namely, that the series-line spectrum of hydrogen, which alone shows the Stark effect, is emitted by the positively charged molecule or atom of hydrogen. The reasoning leading to this deduction may seem rather indirect, but the fact that the above assumptions are the only ones, as I believe, which will explain the presence of the intensity minimum in the Stark effect on the basis of the experimentally verified hypothesis that the emission of light in the path of the canal rays is due to the collision of the canal rays and their offspring with gas molecules, is strong evidence for their correctness. Some other experiments will be performed shortly to test some further deductions from this analysis of the phenomenon.

DISCUSSION OF STRASSER'S RESULTS

Last April in the *Annalen der Physik*,¹ B. Strasser published some results of his painstaking experiments on the Stark effect which are of great interest. May I make a few suggestions as to a possible interpretation of these results along the same line as

¹ *Annalen der Physik*, 31, 890-918, 1910.

in the case of the simple Stark effect in pure hydrogen? Here again one is working rather in the dark, but a suggestion may have value as a working hypothesis even though it later proves to be false.

Strasser has proved the following facts:

1. The intensity of the rest line in the Stark effect for hydrogen canal rays depends on the purity of the hydrogen in the tube. If the gas is sufficiently pure no rest line is obtained.

2. If a definite quantity of another gas is added, the intensity of the rest line is increased and that of the shifted line decreased in proportion to the amount added, so that when a sufficient quantity of the foreign gas is present an intensity minimum is no longer obtained between the two lines.

3. By experimenting with various gases (*N*, *Ar*, *He*) it was found that the effect produced when the partial pressure of the added gas had a certain value was greater the larger the atomic weight of the gas introduced.

From these results it seems evident that the rest line is due to the collision of hydrogen canal rays with molecules of the foreign gas. The larger the number of molecules of the foreign gas present, the greater the number of collisions with them in proportion to the number of collisions with hydrogen molecules which we have assumed produce the displaced line. If we assume that the charged canal rays are not neutralized for the most part by colliding with foreign gas molecules, but, because of the shock of the collision, emit light whose intensity is proportional to the energy of the collision, the Doppler effect due to this light would resemble in general form the curves shown in Fig. 19, though modified by the fact that in this case the hit molecules are larger. Thus we should expect the rest line to be broadened unsymmetrically toward the displaced line as Strasser found. Moreover gases with greater atomic weights may be expected to have larger molecules, hence to exert a greater influence for a given number of molecules by increasing the probability of being struck by the canal rays. The only difficulty is to explain why the hydrogen canal rays are neutralized for the most part when striking hydrogen molecules but not when hitting other molecules. But Strasser has shown that

the Doppler effect produced in the two cases is quite different. It is hard to see how the facts can be explained otherwise.

Strasser also reports that the light from the layer just in front of the cathode shows a weak displaced line with a distinct intensity minimum, whereas immediately behind the cathode a strong, broad displaced line is obtained corresponding to the high velocity of the canal rays. The explanation seems to be that as the canal rays acquire their velocity through the action of the electric field in front of the cathode, and since the potential-gradient is extremely steep right near the cathode, the number of rays having a velocity high enough to produce ionization must increase very rapidly just at the surface of the cathode. This leads to the apparent discontinuity at the surface of the cathode which suggested Strasser and Wien's theory that the canal rays emit light as a result of the electric shock experienced in suddenly passing from a very strong to a weak field—a hypothesis which of course is no longer tenable.

Strasser's observation that the hydrogen lines persist to a greater distance from the cathode than the lines of a foreign gas may be interpreted to show that the minimum energy required to produce the emission of light is greater in the case of the foreign gas than in the case of hydrogen. Finally Strasser reports that the spectrum of the light from the path of the canal rays viewed normal to their velocity shows the same broadening of the lines whether there is a foreign gas present or not. The computation of the effect to be expected on the basis of the five assumptions made above is extremely laborious, but a first approximation shows a fair qualitative agreement with this experimental result. Strasser does not publish quantitative data as to the amount of the broadening. His more recent results for hydrogen canal rays in nitrogen gas¹ can obviously be explained in the same way as these earlier ones and merely add to the evidence in support of this analysis of the phenomenon.

CONDUCTION OF HEAT BY A GAS AT LOW PRESSURE

The curves shown in Figs. 5 and 6 give some idea of the relative importance of convection and of radiation in the cooling of the

¹ *Annalen der Physik*, **32**, 1107, 1910.

cone. Curve XIV was made with the pressure in the canal chamber about twenty times as great as when curve XXI was taken. From the derivative curves (Fig. 6) it appears that the energy being received by the cone per second in the first case is even greater than in the second case yet the maximum temperature reached in the first case is only one-sixth of that reached in the second case. By differentiating equation (3) we get

$$c\left(\frac{d^2x}{dt^2}\right)_+ = -(A+B+C)\left(\frac{dx}{dt}\right)_+ \quad (5)$$

since the time derivatives of z and y are negligible in comparison with $\frac{dx}{dt}$ for the cooling part of the curve. This enables a rough determination of $(A+B+C)$ to be made. The results show a surprisingly good agreement. The value increases from 0.00010 calories per second at a pressure of 0.005 mm to .00065 calories per second at a pressure of 0.1 mm of mercury—showing that at the higher pressure convection is at least ten and probably twenty times as important as radiation in cooling the cone. The conduction coefficients computed from these data are about 0.00017 and 0.000013 calories per second per degree C. per cm³ for pressures of 0.1 mm and 0.005 mm respectively. By comparison with the coefficient for hydrogen at ordinary pressures, which is given by Meyer as 0.00040, it is seen that the coefficient must be nearly constant until pressures below 1 mm of mercury are reached.

PROPORTION OF NEUTRAL CANAL RAYS

From the energy-flux of the canal rays it is possible to compute the lower limit to the number striking the cone per second by dividing the energy-flux by the energy a singly charged molecule would have if acted on by the whole cathode-fall of potential. Curve I of Fig. 24 was thus computed from the curve in Fig. 8. A lower limit to the number charged could be obtained by dividing the current carried by the rays to the cone by the known value of e . Thus curve II was computed. The increase in the proportion of neutral rays with the increase in the cathode-fall of potential is to be expected, I think, for the particular form of tube which was used, though it might not be true for another tube.

DESCRIPTION OF APPARATUS (*Continued*)

To admit the gas at a uniform rate a capillary was used. The best dimensions were determined by the use of Knudsen's formula.¹

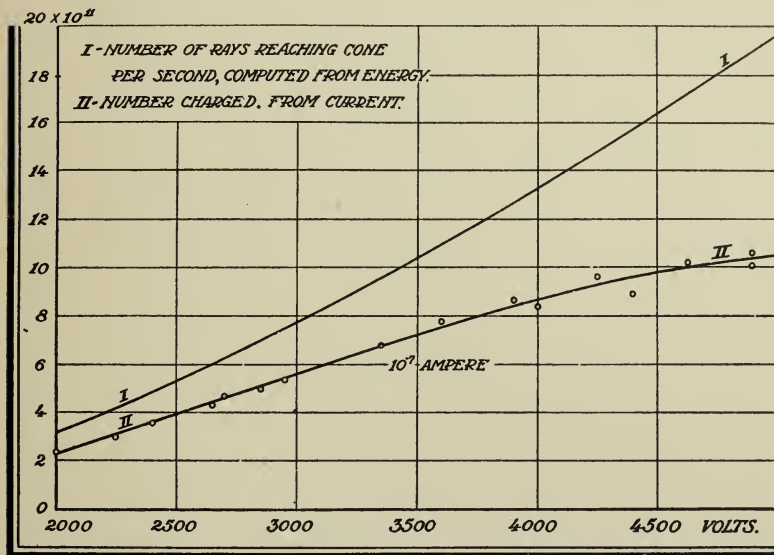


FIG. 24

A number of capillaries were drawn and left attached to tubes of larger bore. The method employed to measure them is illustrated in Fig. 25. Mercury was forced part way into the capillary from the larger tube by means of a rubber bulb, and the capillary was



FIG. 25.—Calibration of a capillary tube

calibrated by noting the corresponding distances moved by the two ends of the mercury thread. From the diameter of the larger tube that of the smaller could thus be readily calculated.

To remove the mercury vapor from the gas, it was passed first

¹ *Annalen der Physik*, 28, 75, 1909.

through flowers of sulphur and copper filings to remove the sulphur vapor, then through a coil kept immersed in a freezing mixture of solid CO_2 and ether. All traces of the mercury lines disappeared from the discharge tube with this arrangement.

To secure a continuous discharge the following arrangement was hit upon (see Fig. 4). An induction coil was used to keep a capacity charged by periodically sparking across a point-and-plane gap made sufficiently long to prevent back sparking. This capacity of 0.3 microfarad was allowed to discharge slowly and continuously through a resistance of several hundred thousand ohms (made by rubbing graphite on a plate of ground glass) which was placed in series with the discharge tube. No discontinuities in the discharge could be observed with a rotating mirror. After the induction coil was stopped the luminous discharge would continue sometimes for over a minute. The discharge therefore was probably quite continuous. If the induction coil was connected to the tube directly, producing a discontinuous discharge, a spark-gap of one centimeter would short-circuit the tube if placed in parallel with it, whereas if the condenser set-up was used with the same pressure in the tube, 5,000 volts would maintain a continuous discharge through the tube. Other evidence was secured tending to show that a continuous discharge is an unstable phenomenon unless very special precautions are taken; and that the mean discharge potential is always greater in the case of the discontinuous discharge than in the case of the continuous, though much less than the maximum value reached by the oscillating potential-difference in the former case. A rotating mirror should always be used to test the continuity of the discharge in the case for any quantitative experiments involving the discharge of electricity through gases.

My thanks are due to Professor A. G. Webster for his interest and encouragement during my stay at Clark University, where the above theory was developed.

MADISON, WIS.

December 28, 1910

ADDENDUM

By a strange mischance I failed to see, until the above was in type, an article by Professor W. Wien¹ in which he gives results of his measurements of the luminosity of a canal-ray beam at various pressures. He finds that the luminosity increases with the pressure, in qualitative agreement with my observations, but a quantitative comparison would be difficult because of the complexity of the conditions existing with his form of apparatus. His interpretation of the results is quite different from mine.

¹ *Annalen der Physik*, 30, 349-368, 1909.



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